Computation of flux, density and pressure after trajectory simulation using Delcourt's model.

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POLAR WIND RELEASE, CONSTANT OUTFLOW (IONOS)

Initial conditions.

energy:[50..100] eV
mlat: [55..90] degrees (northern)
[–55..90] degrees (southern)
pitch: [170..180] degrees (northern)
[0..10] degrees (southern)
phase: [0..360] degrees
mlt: [0..24] hours
R: 4.0 RE

For $E_1 = 50$ eV, $v_1 = 97.85246$ km/s
For $E_2 = 100$ eV, $v_2 = 138.384$ km/s

For static fields release, energy, mlat, pitch, phase, and mlt vary uniformly.

For dynamic fields, release is uniform in velocity space, i.e.,
Phase varies uniformly.
Energy is uniform in $E^{3/2}$, i.e., $E^{3/2}$ is uniformly random.
Pitch is uniform in cos(pitch).

Computations in IDL plotting program.

For each bin a particle goes through, read energy *E* (in eV) and transit time *T*.
Then compute for that particle and bin:

$$\text{flux } (\text{cm}^{2}\text{s}^{-1}\text{sr}^{-1}\text{keV}^{-1}) = 1.833847\left(\frac{E}{1\times 10^{3}}\right) psd\ (\text{km}^{-6}\text{s}^{3})$$

For static fields, (non-uniform physical space, non-uniform velocity space),

$$n_{ij}\ (\text{km}^{-3}) = \frac{1}{\text{Volume}}\, psd\, \tilde{A} V \text{v} \frac{\sqrt{E_{init}}\sin\theta\cos\lambda}{\sum\limits_{\text{all particles}} \sqrt{E_{init}}\sin\theta\cos\lambda} \text{v}_{\text{init}} T$$

For dynamic fields (non-uniform physical space, uniform velocity space),

$$n_{ij}\ (\text{km}^{-3}) = \frac{1}{\text{Volume}}\, psd\, \tilde{A} V\text{v} \frac{\text{T}_{\text{Tot}}}{\Delta t} \frac{\cos\lambda}{\sum\limits_{\text{all particles}} \cos\lambda} \text{v}_{\text{init}} T$$

where

n_{ij} = Density contributed by that particle
Volume = 1 RE^3 (Bin volume)
$psd = 6\text{x}10^9\ \text{km}^{-6} s^3$
N_T = Total number of particles released
v_{init} in RE/s
θ = Initial pitch angle
λ = Initial magnetic latitude
T = Transit time
T_{Tot} = Total simulation time [dynamic fields]
Δt = Frame time length [dynamic fields]
$\tilde{A} = 18.18\ \text{RE}^2$
$V\text{v} = 109{,}018\ \dfrac{\text{km}^3}{\text{s}^3}$

POLAR WIND WITH CAPS INITIAL CONDITIONS, DYNAMIC FIELDS (IOCAP)

Hydrogen ions were released uniformly from 60 to 90 degrees latitude around both poles at an altitude of 1000 km.

The CAPS files contain the following values on a grid. Interpolate to get values at arbitrary points. See CAPS-computations.rtf in website to see how to compute Eth [eV], Phi [V] and E|| [eV].

Compute initEPerp as a (uniform) random number between 0 and Eth. Compute initial perpendicular velocity from initEPerp.

minEParal = E// -Eth/2
maxEParal = E// + Eth/2

Compute sqrtEParal as a (uniform) random number between sqrt(minEParal) and sqrt(maxEParal). Compute initial parallel velocity from sqrtEParal.

The release is uniform in velocity space: the parallel velocity varies uniformly, and the square of the perpendicular velocity varies uniformly. The phase varies uniformly from 0 to 360 deg. The pitch is calculated from the parallel and perpendicular velocities.

Computations in IDL plotting program.

For each particle, compute SZA (Solar Zenith Angle), corresponding to the location the particle was released from. Compute flux:

0 < SZA < 90:	flux at 1000 km = 2x10^8	[cm-2 s-1]
90 < SZA < 110:	fluxat1000km=2x10^(8 –((SZA–90)/20)*2.5)	[cm-2s-1]
110 < SZA < 180:	flux at 1000 km = 2x10^5.5	[cm-2 s-1]

Compute for each particle and bin:

$$n\ (\mathrm{cm}^{-3}) = \frac{1}{\mathrm{Volume}}\mathrm{flux}\left|\cos(\theta)\right|\frac{\tilde{A}}{N_T}T\ \frac{\mathrm{T_{Tot}}}{\Delta \mathrm{t}}$$

where

n = Density contributed by that particle
$\mathrm{Volume} = 1\ \mathrm{RE}^3$ (Bin volume)
N_T = Total number of particles released
$\mathrm{v}_{\mathrm{init}}$ in RE/s
θ = Initial pitch angle
T = Transit time
$\tilde{A} = 2.25289\ \mathrm{RE}^2$
T_{Tot} = Total simulation time
$\Delta \mathrm{t}$ = Frame time length

SOLAR WIND RELEASE, STATIC FIELDS.

Initial conditions

Energy: [0..10] eV
pitch: [0..180] degrees
phase: [0..360] degrees

The release is uniform in velocity space, i.e.,
Phase varies uniformly.
$E^{3/2}$Energy is uniform in , i.e., $E^{3/2}$ is uniformly random.
Pitch is uniform in cos(pitch).

Solar wind particles were released in three batches.

Batch 1 contained 400,000 particles released uniformly on a large rectangle:

X=15 RE
Y=[-30..30] RE
Z=[-30..30] RE

Batch 2 contained 400,000 particles released uniformly on a small rectangle:

X=15 RE
Y=[-10..10]
RE Z=[-15..15] RE

Batch 3 contained 2.2 million particles released uniformly on a small rectangle:

X=15 RE
Y=[-10..10] RE
Z=[-10..10] RE

To reduce batch 3 file sizes, the files were reduced in such a way that each bin contains at most 100 particles. For each bin, only the information for the first 100 particles that go through it is kept in the files.

Computations in IDL plotting program.

For each bin a particle goes through, read energy *E* (in eV) and transit time *T*. Then compute for that particle and bin:

$$psd = 3.381\text{x}10^7\ \text{n}\left(\frac{1}{2\pi kT}\right)^{\frac{3}{2}} \exp\left(-\frac{E_{\text{init}}}{kT}\right)$$

where

$\text{n} = 6.49\ \text{cm}^{-3}$ (density at x=15 RE, y=0, z=0 in DENSITY.0680)
$kT = 4.9\text{x}10^{-3}\text{keV}$ (temperature at x=15 RE, y=0, z=0 in TEMP.0680)
E_{init} in keV

$$\text{flux}\ (\text{cm}^{2}\text{s}^{-1}\text{sr}^{-1}\text{keV}^{-1}) = 1.833847\left(\frac{E}{1\times 10^{3}}\right) psd\ (\text{km}^{-6}\text{s}^{3})$$

For solar wind release containing only batch 1,

$$n\ (\text{km}^{-3}) = \frac{1}{\text{Volume}}\, psd \frac{\tilde{A}\ V\text{v}}{N_T} \text{v}_{init} T$$

For solar wind release containing two batches (1 and 2 or 1 and 3),

$$n\ (\text{km}^{-3}) = \frac{1}{\text{Volume}} \frac{psd\ V\text{v}\ \text{v}_{init} T}{Ni}$$

where

n = Density contributed by that particle
$\text{Volume} = 1\ \text{RE}^3$ (Bin volume)
N_T = Total number of particles released
v_{init} in RE/s
T = Transit time
$\tilde{A} = \Delta y \Delta z = 3600\ \text{RE}^2$
$Vv = 351{,}251 \dfrac{\text{km}^3}{\text{sec}^3}$

If batches 1 and 2 are used:

$Ni = \dfrac{400{,}000}{3600} + \dfrac{400{,}000}{600}$ if particle originated inside the small rectangle

$Ni = \dfrac{400{,}000}{3600}$ if particle originated outside the small rectangle

If batches 1 and 3 are used:

If the bin reached 100 particles, do the computations as if the simulation had stopped when the bin reached 100 particles, i.e., take to be the number of particles needed to fill the bin with 100 particles, which is the highest particle number that goes through the bin. If the bin did not reach 100 particles, do the computations using the total number of particles.

Then, Ni is computed as follows:

If bin particles < 100 then
  If particle origin inside small rectangle
$$Ni = \frac{400{,}000}{3600} + \frac{2{,}200{,}000}{400}$$
  If particle origin outside small rectangle
$$Ni = \frac{400{,}000}{3600}$$

If bin particles = 100 then
  If origin inside small rectangle
    If highest particle <= 400,000 then
$$Ni = \frac{highest}{3600}$$
    If highest > 400,000
$$Ni = \frac{400{,}000}{3600} + \frac{highest - 400{,}000}{400}$$

If particle origin outside small rectangle

$$Ni = \frac{400{,}000}{3600}$$

For ease of implementation, the following equivalent algorithm was used:

If bin particles < 100 then

If particle origin inside small rectangle

$$Ni = \frac{400{,}000}{3600} + \frac{2{,}200{,}000}{400}$$

If particle origin outside small rectangle

$$Ni = \frac{400{,}000}{3600}$$

If bin particles = 100 then

If highest particle <= 400,000 then

$$Ni = \frac{highest}{3600}$$

If highest > 400,000

If particle origin inside small rectangle

$$Ni = \frac{400{,}000}{3600} + \frac{highest - 400{,}000}{400}$$

If particle origin outside small rectangle

$$Ni = \frac{400{,}000}{3600}$$

SOLAR WIND RELEASE, DYNAMIC FIELDS (SOLRE)

Initial conditions.

energy:[0..10] eV
pitch: [0..180] degrees
phase: [0..360] degrees

The release is uniform in velocity space, i.e.,
Phase varies uniformly.
$E^{3/2}$Energy is uniform in , i.e., $E^{3/2}$ is uniformly random.
Pitch is uniform in cos(pitch).

Solar wind particles were released in two batches.

Batch 1 contained 20 million particles released uniformly on a large rectangle:

X=15 RE

Y=[-30..30] RE
Z=[-30..30] RE

Batch 2 contained 80 million particles released uniformly on a small rectangle:

X=15 RE
Y=[15..1] RE
Z=[15..15]RE

To reduce file sizes, each bin/1-minute slice contains at most 5 particles. For each bin/1minute slice, only the information for the first 5 particles that go through it is kept in the files.

Computations in IDL plotting program.

For each bin a particle goes through, read energy E (in eV) and transit time T.
Then compute for that particle and bin:

$$psd = 3.381\text{x}10^7\ \text{n}\left(\frac{1}{2\pi kT}\right)^{\frac{3}{2}}\exp\left(-\frac{E_{\text{init}}}{kT}\right)$$

where

$\text{n} = 6.49\ \text{cm}^{-3}$ (density at x=15 RE, y=0, z=0 in DENSITY.0680)
$kT = 4.9\text{x}10^{-3}\text{keV}$ (temperature at x=15 RE, y=0, z=0 in TEMP.0680)
E_{init} in keV

$$\text{flux}\ (\text{cm}^2\text{s}^{-1}\text{sr}^{-1}\text{keV}^{-1}) = 1.833847\left(\frac{E}{1\times10^3}\right)psd\ (\text{km}^{-6}\text{s}^3)$$

The density and pressure are computed separately for each bin/1-minute slice. The density and pressure for each bin/frame is the average of its bin/1-minute slice components.

$$n\ (\text{km}^{-3}) = \frac{1}{\text{Volume}}\frac{psd\ Vv\ \text{v}_{init}T}{Ni}\ \text{Ns}$$

where

n = Density contributed by that particle
Volume = 1 RE^3 (Bin volume)
N_T = Total number of particles released
v_{init} in RE/s
T = Transit time
Ns=Simulation time in minutes

$$Vv = 351{,}251\frac{\text{km}^3}{\text{sec}^3}$$

If the bin/1-minute slice reached 5 particles, compute Ni as if the simulation had stopped when the bin reached 5 particles, i.e., take to be the number of particles needed to fill the bin/1-minute slice with 5 particles, which is the highest particle number that goes through the bin. If the bin did not reach 5 particles, compute Ni using the total number of particles.

Then, Ni is computed as follows:

If bin particles < 5 then
  If particle origin inside small rectangle

$$Ni = \frac{\text{Batch 1 count}}{3600} + \frac{\text{Batch 2 count}}{900}$$

  If particle origin outside small rectangle

$$Ni = \frac{\text{Batch 1 count}}{3600}$$

If bin particles = 5 then
  If origin inside small rectangle
    If highest particle <= Batch 1 count then

$$Ni = \frac{highest}{3600}$$

    If highest > Batch 1 count

$$Ni = \frac{\text{Batch 1 count}}{3600} + \frac{highest - \text{Batch 1 count}}{900}$$

If particle origin outside small rectangle

$$Ni = \frac{\text{Batch 1 count}}{3600}$$

For ease of implementation, the following equivalent algorithm was used:

If bin particles < 5 then
  If particle origin inside small rectangle

$$Ni = \frac{\text{Batch 1 count}}{3600} + \frac{\text{Batch 2 count}}{900}$$

  If particle origin outside small rectangle

$$Ni = \frac{\text{Batch 1 count}}{3600}$$

If bin particles = 5 then
  If highest particle <= Batch 1 count then

$$Ni = \frac{highest}{3600}$$

If highest > Batch 1 count
  If particle origin inside small rectangle

$$Ni = \frac{\text{Batch 1 count}}{3600} + \frac{\textit{highest} - \text{Batch 1 count}}{900}$$

If particle origin outside small rectangle

$$Ni = \frac{\text{Batch 1 count}}{3600}$$

AURORAL WIND RELEASE WITH CONSTANT OUTFLOW (AUROR)

Oxygen ions were released uniformly in a band around both poles, approximated by the area between a pair of circles. In the north side, the most poleward circle has a diameter of 30 deg and centered at the pole. Thus the circle has latitude of 75 deg all around. The equatorward circle has a diameter of 55 deg, offset by 2.5 deg toward midnight so it has latitude 65 deg at noon and 60 deg at midnight.

The northern equatorward circle was approximated by letting phi vary from 0 to 180, and

$$\theta = 25 + \frac{5}{180}\phi$$

This approximation deviates from the true theta by a maximum of 0.6 degrees.

The southern circles are similar.

The auroral zones are divided into night and day sides by the terminator taken at equinox at 06 and 18 hrs local time.

The dayside ovals emit O+ at altitude 1000km, with a flux of 1e9 cm-2 s-1: ns = 1e3 cm-3; vs = 1e6 cm s-1; vth = 1e6 cm s-1, so the parallel velocity range is 0.1e6 to 2e6 cm s-1 and the perpendicular velocity range is 0 to 1e6 cm s-1

The nightside ovals emit O+ at 1000 km, with a flux of 1e8 cm-2 s-1: ns = 1e1 cm-3; vs = 1e7 cm s-1; vth = 1e7 cm s-1, so the parallel velocity range is 0.1e7 to 2e7 cm s-1 and the perpendicular velocity range is 0 to 1e7 cm s-1

To make the release uniform in velocity space, the parallel velocity varies uniformly, and the square of the perpendicular velocity varies uniformly. The phase varies uniformly from 0 to 360 deg. The pitch can be calculated from the parallel and perpendicular velocities.

Computations in IDL plotting program.

For each bin a particle goes through, read energy *E* (in eV) and transit time *T*.

Then compute for that particle and bin:

$$psd = 3.381 \times 10^{7}\ \text{ns} \left(\frac{16}{2\pi kT}\right)^{\frac{3}{2}} \exp\left(-\frac{E_{\text{init}}}{kT}\right)$$

where

$ns = 1 \times 10^{3}\ \text{cm}^{-3}$ [day side]

$ns = 1 \times 10^{1}\ \text{cm}^{-3}$ [night side]

$kT = 4.9 \times 10^{-3}\,\text{keV}$ (temperature at x=15 RE, y=0, z=0 in TEMP.0680)

E_{init} in keV

$$\text{flux}\ (\text{cm}^{2}\text{s}^{-1}\text{sr}^{-1}\text{keV}^{-1}) = 1.833847\left(\frac{E}{1 \times 10^{3}}\right) psd\ (\text{km}^{-6}\text{s}^{3})$$

$$n\ (\text{cm}^{-3}) = \frac{1}{\text{Volume}} \left|\cos(\theta)\right| ns \frac{\tilde{A}}{N_T} \text{v}_{init} T \ \text{[static fields]}$$

$$n\ (\text{cm}^{-3}) = \frac{1}{\text{Volume}} \left|\cos(\theta)\right| ns \frac{\tilde{A}}{N_T} \text{v}_{init} T \ \frac{\text{T}_{\text{Tot}}}{\Delta \text{t}} \ \text{[dynamic fields]}$$

where

n = Density contributed by that particle

Volume = 1 RE^3 (Bin volume)

N_T = Total number of particles released

v_{init} in RE/s

θ = Initial pitch angle

T = Transit time

$ns = 1 \times 10^{3}\ \text{cm}^{-3}$ [day side]

$ns = 1 \times 10^{1}\ \text{cm}^{-3}$ [night side]

$\tilde{A} = 1.333\ \text{RE}^2$

T_{Tot} = Total simulation time [dynamic fields]

Δt = Frame time length [dynamic fields]

AURORAL WIND WITH CAPS INITIAL CONDITIONS (AUCAP)

Oxygen ions were released uniformly from 60 to 90 degrees latitude around both poles at an altitude of 1000 km.

The CAPS files contain the following values on a grid. Interpolate to get values at arbitrary points. See CAPS-computations.rtf in website to see how to compute Eth [eV], Phi [V] and E|| [eV].

Compute initEPerp as a (uniform) random number between 0 and Eth
Compute initial perpendicular velocity from initEPerp.

minEParal = E// - Eth/2
maxEParal = E// + Eth/2

Compute sqrtEParal as a (uniform) random number between sqrt(minEParal) and sqrt(maxEParal). Compute initial parallel velocity from sqrtEParal.

The release is uniform in velocity space: the parallel velocity varies uniformly, and the square of the perpendicular velocity varies uniformly. The phase varies uniformly from 0 to 360 deg. The pitch is calculated from the parallel and perpendicular velocities.

Computations in IDL plotting program.

Read from the CAPS files S (poynting flux) and density corresponding to the release location (and time for dynamic fields). See CAPS-computations.rtf in website to see how to compute flux (called NiV// in that document).

Compute for each particle and bin:

$$n\ (\mathrm{cm}^{-3}) = \frac{1}{\mathrm{Volume}}\left|\cos(\theta)\right| ns \frac{\tilde{A}}{N_T} \mathrm{v}_{init} T \ \text{[static fields]}$$

$$n\ (\mathrm{cm}^{-3}) = \frac{1}{\mathrm{Volume}}\mathrm{flux}\left|\cos(\theta)\right| \frac{\tilde{A}}{N_T} T \ \frac{\mathrm{T_{Tot}}}{\Delta \mathrm{t}} \ \text{[dynamic fields]}$$

where
n = Density contributed by that particle
$\mathrm{Volume} = 1\ \mathrm{RE}^3$ (Bin volume)
N_T = Total number of particles released
$\mathrm{v}_{\mathrm{init}}$ in RE/s
θ = Initial pitch angle
T = Transit time
$\tilde{A} = 2.25289\ \mathrm{RE}^2$
T_{Tot} = Total simulation time [dynamic fields]
$\Delta \mathrm{t}$ = Frame time length [dynamic fields]

PLASMASPHERE RELEASE, EQUATORIAL (PLASM)

Hydrogen ions were released on the equatorial plane (Z=0) on an approximate circle of about 8 RE. Mei-Ching provided the file NBz-SBz.plsbny which defined the exact shape of the release points. The file contains a grid of local times and for each time step/local time the radius of the release circle and the nb density value. Linear interpolation was used for points not on the grid.

energy:[0.5..1.5] eV
mlat: 0 degrees (equatorial plane)

pitch: [0..180] degrees
phase: [0..360] degrees
mlt: [0..24] hours
R: From NBz-SBz.plsbny

Energy, pitch, phase and mlt vary uniformly.

Computations in IDL plotting program.

Read from NBz-SBz.plsnby the density nb corresponding to the release location and time. Find the average (for all local times) at time of release. Define H=2.0 RE.

Compute

$$\tilde{A} = 2\pi \bar{R} H$$

Compute for each particle and bin:

$$n\,(\mathrm{cm}^{-3}) = \frac{1}{\mathrm{Volume}}\, nb \frac{\tilde{A}}{N_T} \mathrm{v}_{init} T \,\frac{\mathrm{T_{Tot}}}{\Delta \mathrm{t}}$$

where

n = Density contributed by that particle
$\mathrm{Volume} = 1\ \mathrm{RE}^3$ (Bin volume)
N_T = Total number of particles released
$\mathrm{v}_{\mathrm{init}}$ in RE/s
T = Transit time
nb from NBz - SBz.plsbny
T_{Tot} = Total simulation time
$\Delta\mathrm{t}$ = Frame time length

PLASMASPHERE RELEASE ALONG FIELD LINES (PLSFH)

Hydrogen ions were released along the field lines that cross the equator at 6.6 RE or at the last closed field line, which one comes first. First a random mlt is chosen, then Ro is determined by interpolation with data in the .plsbny file. The field line is traced in both directions stopping at an altitude of 3.2 RE. Once the field line is traced a point on the line is chosen at random.

energy:[0.5..1.5] eV
mlat: Determined by the randomly chosen point.
pitch: [0..180] degrees
phase: [0..360] degrees
mlt: [0..24] hours
R: Determined by the randomly chosen point.

Energy, pitch, phase and mlt vary uniformly.

Computations in IDL plotting program.

The initial velocity output from the Fortran program was taken from the MHD velocity file (instead v_{init} of Delcourt's model as in previous releases.)

Read from .3Dplsp file the density ns corresponding to the release location and time.

Compute for each particle and bin:

$$n\ (\mathrm{cm}^{-3}) = \frac{1}{\mathrm{Volume}} ns \frac{\tilde{A}}{N_T} \mathrm{v}_{init} \lambda T \ \frac{\mathrm{T_{Tot}}}{\Delta \mathrm{t}}$$

where

n = Density contributed by that particle
$\mathrm{Volume} = 1\ \mathrm{RE}^3$ (Bin volume)
N_T = Total number of particles released
$\mathrm{v}_{\mathrm{init}}$ in RE/s
λ = Angle between initial velocity vector and the normal to the dipole field surface
T = Transit time
ns from .3Dplsp file
T_{Tot} = Total simulation time
Δt = Frame time length
$\tilde{A}$ = release area determined by the dipole field lines that cross the equator at R = Ro and stop at an altitude of 3.2 RE.

FLUX COMPUTATION

The initial conditions from the cluster simulation were used to compute the total flux out of the release area. The flux is calculated for each 4-minute time slice in the whole simulation span. Each 4-minute slice is called a "frame". The cluster run used the magnetic fields provided by Steve Slinker (NRL), but for this flux computation a dipole field approximation was used to find the normal to the magnetic field at each particle's initial position. Note that if the normal points inward then cos(angle) is less than zero and the flux is subtracted from the total. The exact values obtained from this algorithm will depend on the random initial conditions, but the flux value should converge as more particles are read from the input file.

ABar is the release surface area
frame_count = slice time length * (particles used in computation/total simulation time)
dA = ABar / frame_count
For each particle in cluster data file
    Read initial conditions
    Compute frame number based on release time
    Compute dipole field vector at particle's initial position
    Compute normal to release surface

Compute angle between initial velocity vector and normal to release surface
Interpolate ns value from crcmdlfm.3Dplsp file
Compute flux = ns * initial velocity * cos(angle) * dA
Add flux to current frame total flux
Endfor

PRESSURE AND ENERGY

The following applies to all releases.

Pressure is given by

$$P_{ij} = \frac{2n_{ij}\overline{E}}{3} \left(\frac{\text{Newton}}{\text{m}^2}\right)$$

where

n_{ij} = density contributed by particle in m^{-3}

$\overline{E}$ = average energy for particle in bin in Joules

The flux for each bin is the average of all the fluxes from each particle.

The density for each bin is the sum of all the contributions from each particle. The pressure for each bin is the sum of all the contributions from each particle. After density and pressure are computed for each bin, the energy for a bin is

$$E = \frac{3}{2}\frac{P}{n}$$

where

P = Total pressure in bin

n = Total density in bin

APPENDIX

The data file contains two kinds of lines: particle lines and bin lines. The particle line contains initial conditions for a given particle. The bin line contains the bin coordinates and particle state at that bin. Each particle line is followed by many bin lines.

$\tilde{A}$ and $V\mathrm{v}$ computations for **polar wind release.**

$$\tilde{A} = 2\int_{\theta=0^\circ}^{\theta=35^\circ}\int_{\phi=0}^{\phi=2\pi} \mathrm{R}^2 \sin\theta d\phi d\theta = 18.18 \text{ RE}^2$$

$$V\mathrm{v} = \iiint d^3\mathrm{v} = \int_{\mathrm{v1}}^{\mathrm{v2}}\int_{\theta=0^\circ}^{\theta=10^\circ}\int_{\phi=0}^{\phi=2\pi} \mathrm{v}^2 \sin\theta d\phi d\theta d\mathrm{v} = 109{,}018 \ \frac{\mathrm{km}^3}{\mathrm{s}^3}$$

$\tilde{A}$ and $V\mathrm{v}$ computations for solar wind release.

$$\tilde{A} = \Delta y \Delta z = 3600 \text{ RE}^2$$

$$Vv = \frac{4}{3}\pi(v_2^3 - v_1^3)$$

$$E\min = 0\mathrm{ev} \Rightarrow v_1 = 0\frac{\mathrm{km}}{\mathrm{sec}}$$

$$E\max = 10\mathrm{ev} \Rightarrow v_2 = 43.77\frac{\mathrm{km}}{\mathrm{sec}}$$

$$Vv = 351{,}251\frac{\mathrm{km}^3}{\mathrm{sec}^3}$$

$\tilde{A}$ computations for auroral wind release.

$$\tilde{A} = 4\int_{\phi=0^\circ}^{180^\circ}\int_{\theta=15}^{25+\frac{5}{180}\phi} \mathrm{R}^2 \sin\theta d\theta d\phi = 1.333 \text{ RE}^2$$

Derivation of *n* for solar wind release (released uniformly in phase space) (written by Mei-Ching):

For each particle released in the ionosphere,

$$dN = psd\, d^3\mathrm{v}\, d^3r$$

If $j = flux = particle / s$,

$$j = \frac{dN}{\Delta t} = psd\, d^3\mathrm{v}\, dA \frac{\Delta l}{\Delta t},$$

$$j = psd \frac{\iiint \iint \mathrm{d}^3\mathrm{v}\, dA}{\mathrm{N_T}} \mathrm{v_{init}}$$

$$j = psd \frac{\tilde{A}\, V\mathrm{v}}{N_T} \mathrm{v_{init}}$$

In the magnetosphere in a volume, the density, *n*, contribution by that particle is

$$n = \frac{1}{Volume} jT$$

$$n = \frac{1}{Volume} psd \frac{\tilde{A}\, V\mathrm{v}}{N_T} \mathrm{v}_{init} T$$

where $Volume = 1RE^3$
T = transit time or residence time

Derivation of *n* for ionsopheric release (uniformly in E, mlat (λ), and pitch angle θ.) (written by Mei-Ching):

$$d^3VdA = (\mathrm{v}^2 \sin\theta d\mathrm{v}\, d\theta d\phi)(R^2 \sin\theta_c d\theta_c d\phi_c)$$

$$= const\ \sqrt{E_{\mathrm{init}}}\sin\theta_p \cos\lambda, \qquad \text{since } dE_{\mathrm{init}} = m\mathrm{v}d\mathrm{v},\ \sin\theta_{\mathrm{c}} = \cos\lambda$$

and

$$\iiint \iint d^3\mathrm{v}dA = \sum_{\text{all particles}} d^3\mathrm{v}dA = const \sum_{\text{all particles}} \sqrt{E_{init}}\sin\theta\cos\lambda$$

$$\therefore\ const = \frac{\iiint \iint d^3\mathrm{v}dA}{\sum_{\text{all particles}} \sqrt{E_{init}}\sin\theta\cos\lambda}$$

$$= \frac{\tilde{A}V\mathrm{v}}{\sum_{\text{all particles}} \sqrt{E_{init}}\sin\theta\cos\lambda}$$

$$\therefore\ n = \frac{1}{Volume} psd\, \tilde{A}V\mathrm{v} \frac{\sqrt{E_{init}}\sin\theta\cos\lambda}{\sum_{\text{all particles}} \sqrt{E_{init}}\sin\theta\cos\lambda} \mathrm{v}_{\mathrm{init}}T$$

The following was in an email sent by Tom Moore:

To calculate density, we use the method similar to that used in Delcourt et al. [1989, JGR, p11893]. We divided the entire simulation space into bins of 1 RE^3 volume. For a particle (i) passing through a particular bin (j), the contribution of density of this bin by this particle is:

nij=Fi*Tij/Vj (1)

where Fi is the ion source flux in ion/s, Tij is the residence time of particle i in bin j, and Vj is the volume of bin j, that is 1 RE^3 in our case.

Fi is computed directly from the density and flow of the source plasma across the source boundary.

Fi=ns *vs *dA ; dA=A/NT (2)

Here dA is the area of the source surface allocated to each particle, which is the total area of the source divided by the number of particles emitted, assuming a uniform distribution of particle emission on the source surface, which is assured by randomizing the initial locations. The source number density and flow velocity may be specified, or the product of those two is just as useful, if better known.

Substituting (2) into (1), we have

nij=ns *vs*A**Tij/(Vj*NT) (3)

The density at bin j is just the summation of nij over all particles that are passing through bin j:

nj = Summation in i (nij) (4)

These relations can be applied to any source flowing across a boundary surface. Once densities are calculated, pressure at bin j is given by,

Pj = Summation in i (Pij) and

Pij = 2 * nij * Eij / 3

where Eij is the average energy of particle i in bin j.